Majid Ghayoomi,⁴ and John S. McCartney²

Measurement of Small-Strain Shear Moduli of Partially Saturated Sand During Infiltration in a Geotechnical Centrifuge

ABSTRACT: This paper describes the use of bender elements to measure changes in small strain shear modulus, $G_{\text{max}}$, of sand layers due to the change in degree of saturation during centrifuge tests. The goal of the measurements is to verify that steady-state infiltration is an appropriate technique to control the effective stress in centrifuge physical modeling of partially saturated sands. Specifically, the suitability of infiltration is assessed by checking if the measured values of $G_{\text{max}}$ of partially saturated sand layers follow a similar trend to dry and saturated sand layers when the effective stress is defined from the suction and degree of saturation profiles during steady-state infiltration. Three pairs of bender elements were installed at different depths in a container of Ottawa sand, and the shear wave velocities of the sand were measured during steady-state infiltration into the sand layer. The applied infiltration rate was varied to obtain different uniform distributions of degrees of saturation with depth. Consistent with results from suction-controlled resonant column tests performed on the same sand, the values of $G_{\text{max}}$ measured from the bender element tests varied nonlinearly with degree of saturation with a peak value at a degree of saturation between 0.3 and 0.4. When interpreted in terms of mean effective stress, the values of $G_{\text{max}}$ from the bender element tests on partially saturated sands followed a unique trend consistent with measurements for dry and saturated sands.

KEYWORDS: partially saturated sand, small-strain shear modulus, bender elements, geotechnical centrifuge testing

Introduction

Hardin and Richart (1963) and Hardin and Dmeanvich (1972) observed that the shear modulus of soil is generally constant for shear strain amplitudes less than $10^{-4}$ and is sensitive primarily to the void ratio and effective stress state. Knowledge of the small-strain shear modulus $G_{\text{max}}$ is particularly useful in the prediction of the seismic compression of soil layers during earthquake shaking (Tokimatsu and Seed 1987; Ghayoomi 2011). Ghayoomi et al. (2011) developed a new centrifuge modeling approach to evaluate the seismic compression of partially saturated sand layers in which infiltration was used to control the degree of saturation and matric suction, and thus the effective stress state. Although Ghayoomi et al. (2011) found that trends in seismic compression of sand during steady-state infiltration correlate well with expected changes in $G_{\text{max}}$ estimated from empirical equations, independent measurements of $G_{\text{max}}$ during variations in infiltration rate would help increase confidence in the use of steady-state infiltration for suction control in centrifuge testing.

Bender elements have been used extensively for laboratory measurement of $G_{\text{max}}$ of saturated and dry soils (Shirley and Hampton 1978; Dyvik and Madshus 1985; Bates et al. 1989; Argawal and Ishibashi 1991; Brignoli et al. 1996; Arulnathan et al. 1998; Pennington et al. 2001; Leong et al. 2005) as well as partially saturated soils (Cabarkapa et al. 1999; Ince et al. 2003; Marinho et al. 1995; Alramahi et al. 2007; Ng and Yung 2008; Sawangswiriyawat et al. 2009; Ng et al. 2009). They are typically incorporated into laboratory tests because they induce and measure shear strains with amplitudes less than $10^{-4}$ and are relatively compact in size.

The objective of this paper is to describe how bender elements can be used to verify that infiltration is an appropriate tool to control the effective stress state in centrifuge physical modeling of partially saturated sands. Although bender elements have been used in centrifuge tests in the past (Ismail and Hourani 2003; Lei et al. 2004; Brandenberg et al. 2006; Rammah et al. 2006; Fu et al. 2009; Kim and Kim 2010), their use in partially saturated soils in the centrifuge deserves further investigation. Accordingly; in addition to describing the details of the bender element setup for the centrifuge, the measured values of $G_{\text{max}}$ from the bender elements used in this study are compared with those from suction-controlled resonant column tests on the same sand reported by Khosravi et al. (2010).

Background

Bender elements apply shear waves having small strain magnitudes to soil layers through the use of piezoelectric ceramics. Piezoelectric ceramics deform during application of a voltage difference, or generate a voltage difference when deformed. When a voltage signal is applied to a transmitting element, a shear wave will be induced in the surrounding soil, which can be detected using a receiving element located at a known distance from the transmitting...
element. Shear wave velocity can be measured using a pair of bender elements by measuring the time difference between transmitting and receiving shear waves from one bender element to another. The shear wave velocity, $V_s$, is related to $G_{\text{max}}$ as follows:

$$G_{\text{max}} = \rho V_s^2$$  \hspace{1cm} (1)

where $\rho$ is the total soil density.

Since Shirley and Hampton (1978) introduced bender elements to soil testing, they have been incorporated into many conventional geotechnical tests, including triaxial tests (Bates 1989; Brignoli et al. 1996; and Pennington et al. 2001), resonant column tests (Dyvik and Madshus 1985), oedometer tests (Dyvik and Madshus 1985; Kawaguchi et al. 2001), direct simple shear tests (Dyvik and Madshus 1985), true-triaxial tests (Agarwal and Ishibashi 1991), and large container tests (Blewett et al. 2000). Concerns such as electromagnetic cross-talk, in-plane and out-of-plane directivity, bender resonant frequency, and effects of reflections from boundaries have also been addressed in laboratory and theoretical studies (Anilathan et al. 1998; Greening and Nash, 2004; Lee and Santamarina 2005; Leong et al. 2005). Several studies have used bender elements to study the impacts of matric suction, degree of saturation, and hydraulic hysteresis on the magnitude of $G_{\text{max}}$ for partially saturated soils (Cabarkapa et al. 1999; Inci et al. 2003; Marinho et al. 1995; Alramahi et al. 2007; Ng and Yung 2008; Hoyos et al. 2008; Sawangsuriya et al. 2009, Ng et al. 2009).

Bender elements have been used in centrifuge modeling tests as well, primarily for shear wave velocity tomography to infer zones of densified soil (Ismail and Hourani 2003; Lei et al. 2004; Brandenberg et al. 2006; Rammah et al. 2006; Kim and Kim 2010). Fu et al. (2009) used bender elements in centrifuge tests to evaluate the liquefaction resistance of soils based on their shear wave velocity. Due to difficulties in maintaining a steady partially saturated soil condition in centrifuge testing, the application of bender elements in geotechnical centrifuge tests involving partially saturated soils is not extensive. This barrier has been overcome after Ghayoomi et al. (2011) showed that steady-state infiltration leads to a uniform matric suction distribution suitable for mechanical or seismic testing of partially saturated specimens under controlled conditions.

One of the reasons that careful control of the effective stress state in partially saturated soils is necessary is that $G_{\text{max}}$ is particularly sensitive to the mean effective stress. Specifically, experimental studies on $G_{\text{max}}$ of saturated and dry soils found that $G_{\text{max}}$ and mean effective stress are related in terms of a power function (Hardin and Richart, 1963; Seed and Idriss, 1970; Hardin and Drnevich, 1972). The relationships typically have the following form for sands:

$$G_{\text{max}} = AF(e) \left(\frac{\sigma_\text{m}'}{\sigma_\text{m}}\right)^n$$  \hspace{1cm} (2)

where $\sigma_\text{m}'$ is the mean effective stress, $A$ is a fitting parameter having the same units as $G_{\text{max}}$, $n$ is a dimensionless fitting parameter, and $F(e)$ is a void ratio function. Khosravi and McCartney (2009) observed that Eq. (2) can be extended to partially saturated soils if a single-value effective stress parameter is used to combine the effects of matric suction and total stress on the inter-particle stress.

### Experimental Setup

#### Centrifuge and Container

The 400 g-ton geotechnical centrifuge facility at the Univ. of Colorado Boulder (Ko, 1988) was used for the physical modeling experiments in this investigation. A laminar container developed by Law (1991) having an inside length of 58.42 cm, width of 24.13 cm, and depth of 15.87 cm was used in this study. Although this study did not involve shaking, the laminar container was used to be consistent with seismic compression results from Ghayoomi et al. (2010). This container is suitable for modeling a prototype soil layer having a thickness of 0.1587N meters at a centrifuge acceleration N-times that of earth gravity. This container was modified by Ghayoomi et al. (2011) to permit control of steady-state infiltration and drainage of pore fluid through the soil layer by adding a drainage plate to the bottom of the container and installing a series of spray nozzles suspended above the top of the soil layer. Elevation views of the container are shown in Figs. 1(a), 1(b), and 1(c). A detailed explanation of infiltration setup is available in Ghayoomi et al. (2011). The container was modified by incorporating pedestals to support the transmitting and receiving bender elements.

#### Bender Element Setup and Calibration

Three pairs of bender elements were manufactured for this study to measure the shear wave velocity at different depths. The bender elements were manufactured from a single layer of T226-A4-303X type piezo-ceramic from PiezoSystems, Inc. having a length of 31.75 mm, a width of 12.7 mm, and a thickness of 0.67 mm. After connection of coaxial cables to each side of the piezo-ceramics, several coats of nonconductive polyurethane (M-Coat A) were applied to prevent corrosion or short-circuiting in partially saturated soils. A layer of Silver Print paint obtained from MG Chemicals was then applied to mitigate electrical noise from being transmitted to the piezo-ceramics. After connecting grounding cables to the Silver Print paint, several additional coats of polyurethane were applied to the piezo-ceramic. The completed bender elements were potted with nonconductive epoxy within cylindrical steel pipes (having lengths of 38.1 mm and inside diameters of 15.24 mm) so that the free vibrating length of the exposed piezo-ceramic was 17.78 mm.

A picture of the bender elements and support pedestals during placement of a sand layer is shown in Fig. 2. Each pair of bender elements was installed on the vertical support pedestals so that they would be at depths of 3.51, 7.31, and 11.11 cm from the soil surface. The bender elements were oriented in a vertical direction so that they would generate SH waves. This is important in centrifuge testing so that the self-weight of the soil does not provide a downward reaction on the flat side of the bender elements. A 100-mm square flange at the base of the pedestals was affixed to the base of the container using rubber cement, which helped to absorb any vibrational noise from the centrifuge platform. After aligning the two support pedestals, the horizontal distances (tip-to-tip) between the bender elements were 7.25, 7.55, and 7.75 cm for the top, middle, and bottom pairs of benders, respectively.
A data acquisition system capable of exciting the transmitting bender elements and receiving signals from the receiver bender elements was developed so that the bender elements could be used during flight in the centrifuge. A schematic of the connections in the data acquisition system is shown in Fig. 3. A National Instruments PXI chassis with a NI-8176 DAQ controller was used in this study, similar to that used by Kim and Kim (2010). An 8-channel PXI-6251 module with sampling rate of 1MS/s was used for output signal generation and input data measurement. The maximum amplitude of the generated signal from the PXI-6251 module is ±10 V, which is not sufficient to generate a signal which can be measured by the receiving bender element above the noise of the centrifuge. Accordingly, an EPA-104 linear amplifier manufactured by Piezo Systems, Inc., was used to increase the amplitude of the generated wave to ±200 V. Since the amplifier has only one input and one output channel, a PXI-2527 switch was used to guide the generated wave to the desired transmitting bender element. Possible delays due to presence of switch or cross-talk effects were found to be negligible and less than accuracy of the data acquisition system (i.e., 10⁻⁶ s).

The bender element pairs were calibrated to account for the possible delay due to measurement system. Specifically, a signal was sent from the transmitting bender element to the receiving element when the tips of the two bender elements were in contact. The time delay between the transmitted and received signal was then measured to be 5.4±10⁻⁶, 5.7±10⁻⁶, and 4.7±10⁻⁶ s for the top, middle, and bottom bender element pairs, respectively. This calibration time was subtracted from the measurements made during bender element tests in the soil.

**Infiltration Setup and Calibration**

Similar to Ghayoomi et al. (2011), steady-state infiltration was used to control the degree of saturation and matric suction in the partially saturated soil layer during centrifugation. Specifically, during steady-state infiltration toward a water table at the bottom
of a soil layer, the matric suction and degree of saturation are approximately uniform with height in the soil layer (Zornberg and McCartney 2010). In order to apply infiltration, water from a pressurized storage tank on the centrifuge was sprayed uniformly over the upper surface of the soil layer through a series of six fine-mist spray nozzles. A freely-draining water table was imposed at the bottom of the soil layer using a series of eight drainage ports at the base of the container. Water overflows from the top of a drainage layer of gravel having a uniform particle size of 6.35 mm. A thin geotextile filter was placed on top of the gravel layer to prevent any loss of overlying soil through the drainage ports. Water passing through the drainage ports at the bottom of the specimen was collected in a reservoir to measure the outflow rate.

A schematic of the infiltration and drainage systems is shown in Fig. 4. The changes in volumetric water content in the soil layer were measured using five EC-TM dielectric sensors from Decagon Devices of Pullman, WA. The sensors were arrayed as shown in Fig. 1(a). These sensors were placed in a horizontal orientation, as far as possible from the bender elements, at depths of 1.27, 5.08, 7.30, 8.89, and 12.7 cm from the surface of the soil layer. Since the dielectric sensors are far from the benders and their placement depths are different the benders’ depths, they do not affect the shear wave travel time. A more detailed explanation of the infiltration setup is available in Ghayoomi et al. (2011).

The dielectric sensors were calibrated for use in Ottawa F75 sand using a compaction mold. The volumetric water content was measured using the dielectric sensors and also calculated using the measured density and gravimetric water content. By correlating the measured and actual values, a unique linear equation was found to represent the relationship between the actual volumetric water content and the value measured using the sensors (i.e., $\theta_{\text{actual}} = 0.854 \times \theta_{\text{measured}} + 0.036$).

**Materials**

F-75 Ottawa sand was used as the sand in this study, to be consistent with the seismic compression tests performed by Ghayoomi et al. (2011). A target void ratio of 0.66 was used in all of the tests in this study, which corresponds to a relative density of 45%. When saturated, the sand at this void ratio has a permeability $6 \times 10^{-4}$ cm/s. The geotechnical properties of the F-75 fine silica sand are summarized in Table 1 and the grain size distribution obtained from the sieve analysis test is shown in Fig. 5(a). The Soil Water Retention Curve (SWRC) of F-75 Ottawa sand was measured using a hanging column test with controlled outflow described by McCartney et al. (2008). The SWRC is shown in Fig. 5(b) along with the van Genuchten (1980) SWRC model fitted to the experimental data using least-squares regression. The fitting parameters for the van Genuchten (1980) model are also listed in Fig. 5(a). Even through F-75 Ottawa sand has a relatively uniform particle size distribution, it has sufficient fine soil particles to retain water to suctions up to 10 kPa. The combination of a high saturated permeability and the shape of the SWRC of this sand facilitate the use of infiltration to control the suction and degree of saturation during testing. An infiltration approach would not be appropriate for soils with lower permeability than the smallest infiltration rate that can be applied with an infiltration system, which is why this study is focused on sand.

The SWRC for the sand measured at 1g was used to infer the suction values during centrifugation from the volumetric water content measurements from the dielectric sensors. This is possible because Dell’Avanzi et al. (2004) observed that matric suction values in a model and prototype scale 1:1 for the case that the ratio of the centrifuge arm to the soil layer height is greater than 10. This ratio is equal to 33 for the centrifuge and soil layer evaluated in this study. Further, because capillarity is independent of gravity, it can also be assumed that the SWRC in a model and prototype scale 1:1.

**Procedures**

The sand layer was placed at the target void ratio using dry pluviation (Whitman and Lambe 1988) in lifts of 1 cm atop the geotextile filter and around the bender element pedestals. A flexible plastic membrane glued to the bottom of the container was used to separate the sides of the container from the sand and water so that they do not penetrate into the gaps between the laminar container plates. The final thickness of the sand layer was 15.87 cm. The dielectric sensors were placed in a horizontal orientation in the sand during pluviation.

After placement into the centrifuge, bender element measurements were performed on the dry sand at both 1 and 40 g. Specifically, a 10 V-amplitude sinusoidal pulse wave with an excitation frequency of 10 kHz was generated and was then passed through...
the amplifier to increase the amplitude of the wave to 200 V. This wave was applied to the transmitting bender element, and the shear wave passed through the soil to the receiving bender element. The transmitted signal and a typical received signal are shown in Figs. 6(a) and 6(b), respectively. The travel times were measured several times and the values were averaged. Repeat measurements were consistently within 1.5% of each other. A fast-Fourier transform of the received signal for the window shown in Fig. 6(a) is presented in Fig. 6(c). This figure indicates that the main frequency of the received signal is approximately 5 kHz corresponding to the first deflection while there is also a significant frequency at 3 kHz corresponding to the first peak. The other peaks can be attributed to reflections and centrifuge noise.

In this study, the shear wave travel time was calculated by subtracting the first arrival time of the shear wave recorded by receiver benders from the departure time of the transmitted signal. This approach was selected as Leong et al. (2005) observed that the travel time defined using the first deflection point provides the best match with independent shear wave velocity measurements from ultrasonic pulse tests. Ghayoomi (2011) showed that the calculated $G_{\text{max}}$ based on the shear wave velocity measured using the first deflection point method arrival time selection is more consistent, but because the measured travel times were consistent, the near-field effects affect the shape of the received signal and also the arrival time measurement, so the tip-to-tip length ($L_{tt}$) and signal wavelength ($\lambda$) were selected to minimize these effects. The ratio $L_{tt}/\lambda$ was used as the criterion to evaluate the likelihood of near-field effects. Minimum values of $L_{tt}/\lambda$ reported in the literature to minimize near field effects range from 1.00 (Arulnathan et al. 1998) to 3.33 (Leong et al., 2005). The length $L_{tt}$ in this study was in the range of 7.25 to 7.75 cm and the measured tip to tip travel times ranged from 0.0004 to 0.0008 s. Since $L_{tt}/\lambda = f \times t$ (where $f$ is the wave frequency and $t$ is the tip to tip wave travel time), an excitation frequency of 10 kHz, corresponding to a received wave with a main frequency of 5 kHz, resulted in wavelengths corresponding to $L_{tt}/\lambda$ ratios between 2 to 4. This frequency is consistent with the range of 5 to 25 kHz used by Leong et al. (2005). An excitation frequency of 20 kHz was also evaluated, but because the measured travel times were consistent, the frequency of 10 kHz was used throughout the testing program.

After the bender element measurements were made on the dry sand layer, the centrifuge was stopped. The sand layer was then saturated from the bottom by applying an upward gradient through the soil layer. A de-aired water reservoir outside of the centrifuge was connected to the end of the drainage line while the outflow proportional control valve was open. The dielectric sensors were used to infer changes in volumetric water content during the saturation process. After saturation was completed (i.e., when water reached the sand surface and the dielectric sensors indicated a uniform volumetric water content equal to the porosity), the outflow proportional control valve was open. The dielectric sensors were used to infer changes in volumetric water content during the saturation process. After saturation was completed (i.e., when water reached the sand surface and the dielectric sensors indicated a uniform volumetric water content equal to the porosity), the outflow proportional control valve was closed and the de-aired water reservoir was disconnected from the container.

After a bender element measurement on the saturated sand layer at 1 g, the centrifuge was spun up to 40 g. Another bender element measurement was performed on the saturated sand before the outflow proportional control valve was opened fully while commencing infiltration. The rate of infiltration was controlled using an inflow proportional control valve. The volume of water exiting the pressurized tank was metered using a differential pressure transducer (DPT). In this manner, water was sprayed on surface of the soil until the measured outflow was equal to the applied inflow (i.e., steady-state conditions). Further, steady-state

![FIG. 4—Schematic of the infiltration system and the soil container.](image)

### TABLE 1—Geotechnical properties of F-75 Ottawa sand.

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mineralogy</td>
<td>Quartz, 99.8% SiO₂</td>
</tr>
<tr>
<td>Grain shape</td>
<td>Rounded</td>
</tr>
<tr>
<td>Specific gravity, $G_s$</td>
<td>2.65</td>
</tr>
<tr>
<td>$C_w$</td>
<td>1.71</td>
</tr>
<tr>
<td>$C_v$</td>
<td>1.01</td>
</tr>
<tr>
<td>$e_{\text{min}}$, $e_{\text{max}}$</td>
<td>0.49, 0.80</td>
</tr>
<tr>
<td>$\rho_{\text{min}}$, $\rho_{\text{max}}$</td>
<td>1469, 1781 kg/m³</td>
</tr>
<tr>
<td>$K_{sat}$ at $e = 0.66$</td>
<td>$6 \times 10^{-4}$ cm/s</td>
</tr>
</tbody>
</table>
conditions were also defined when the measured volumetric water content profile was uniform with depth. Bender element measurements were made after reaching steady-state conditions for discharge velocities ranging from $10^{-12}$ to $10^{-5}$ cm/s, which correspond to degrees of saturation ranging from 0.11 to 0.71. After performing a series of tests at 40 g, the centrifuge acceleration was increased to 50 g and several additional infiltration tests and bender element measurements were performed.

**Results**

**Infiltration Results and Estimated Effective Stress Profiles**

Profiles of degree of saturation with height in the sand layer at steady-state conditions under different infiltration rates are shown in Fig. 7(a). These profiles were calculated from the volumetric water content values measured from the dielectric sensors. The degree of saturation values were also converted to matric suction profiles using the SWRC in Fig. 5(b), and are shown in Fig. 7(b). In all of the situations, a nearly uniform degree of saturation was obtained for the different infiltration rates. The location of the bender elements are noted in Figs. 7(a) and 7(b) for reference.

The profiles of degree of saturation and matric suction obtained from solutions to Richards’ equation in the centrifuge presented by Dell’Avanzi et al. (2004) are also included in Figs. 7(a) and 7(b) as solid lines. In general, the dielectric sensor measurements match well with the theoretical predictions, confirming that steady-state infiltration leads to nearly uniform distributions in degree of saturation with height in soil layers in the centrifuge. Although suction is constant with depth during steady-state infiltration, the effective stress is not constant with depth because the total stress increases with depth in proportion to the total unit weight of the sand.
Profiles of effective stress in the sand layer were estimated from the known total stress and matric suction profiles using the approach proposed by Lu et al. (2010). They found that the effective saturation (the degree of saturation weighted between full saturation and residual saturation) was an appropriate estimate for the value of \( v \) in Bishop’s equation for effective stress of partially saturated soils (Bishop 1959). They incorporated the equation for the van Genuchten (1980) SWRC model into Bishop’s equation to predict the relationship between the vertical effective stress \( r_0 \) and matric suction \( w \), as follows:

\[
    r_0 = \frac{r}{C_0} u a(q) + w^{1+\alpha} / C_1^{(N-1)/N} C_0
\]

where \( \alpha \) and \( N \) are parameters for the van Genuchten (1980) SWRC model, \( r \) is the total stress, and \( u_a \) is the pore air pressure (assumed to be zero). Profiles of vertical effective stress estimated using Eq. (3), incorporating the total stress induced by centrifugation and the inferred matric suction profile from Fig. 7(b), are shown in Fig. 7(c). The values of effective stress for the different locations of the bender elements are noted in this figure for reference. This figure indicates that the total stress induced by the self-weight of the sand layer is the primary variable in the definition of the effective stress, but changes in suction due to varying infiltration rates will lead to shifts in the effective stress profile with depth.

**Measurements of \( G_{\text{max}} \) for Partially Saturated Sands**

Pulse wave tests were performed for each pair of bender elements independently after reaching steady-state conditions under a given infiltration rate. The measured shear wave velocities \( (V_s) \) from each pair of bender elements are shown as a function of degree of saturation in Fig. 8(a). In addition, the relationship between total density and degree of saturation is shown in this figure. The measurements of \( V_s \) show a nonlinear trend with degree of saturation while the total density increases linearly. The total density was assumed only to depend on changes in the degree of saturation as Ghayoomi et al. (2011) observed negligible changes in height of similar sand layers during spin-up and infiltration as part of a comprehensive series of seismic compression tests. This figure also shows data for the saturated soils \( (Sr = 1) \), although it should be noted that these data points are representative of hydrostatic conditions while the others are for steady-state infiltration. The positive pore water pressure at the depths of the bender elements lead to a lower effective stress and lower \( G_{\text{max}} \) value than for the tests under steady-state infiltration. The \( V_s \) measurements and total densities are also shown as a function of inferred matric suction Fig. 8(b). A nonlinear trend between total density and suction was obtained because the suction was inferred from the SWRC. The data points from the saturated tests are not included in this figure.

The values of \( G_{\text{max}} \) were calculated from the measured values of \( V_s \) and total density using Eq. (1). The \( G_{\text{max}} \) values calculated for different degrees of saturation at 40 g are shown in Fig. 8(c) for the bender elements at depths of 3.51 and 11.11 cm. Signals were detected from the receiver bender element a depth of 7.31 cm, but the signals were not sufficiently greater than the noise level in the centrifuge. This was attributed to a possible short-circuit in the bender element electrical connections after it was submerged in water. Accordingly, the results from a depth of 7.31 cm...
are not reported. When comparing the results from the bender elements at the two depths, the bender element deeper in the soil profile not only showed a greater magnitude of \( G_{\text{max}} \) due to the greater total stress, but also showed both a greater variation in \( G_{\text{max}} \) with the degree of saturation. Although the peak value of \( G_{\text{max}} \) for the two depths occurred at different values of degree of saturation, the peak values occurred between degrees of saturation of 0.3 and 0.4 (suction values of 4.5 to 5.5 kPa).

The calculated values of \( G_{\text{max}} \) as a function of matric suction are shown in Fig. 8(d). The predicted trend between \( G_{\text{max}} \) and matric suction obtained by combining Eqs. (2) and (3) are also shown in this figure for comparison. The parameters \( A \) and \( n \) were defined using values from Hardin and Richart (1963) reinterpreted in terms of metric units \[ A = 69.77, \ n = 0.5, \ \text{and} \ F(e) = (2.17 - e)/2(1 + e), \] where \( e = 0.66 \). Although there is some scatter in the results especially at lower suction values, the trends and magnitudes of the data and the predicted relationship match reasonably.

Analysis

**Comparison of Bender Element and Resonant Column Results**

Khosravi et al. (2010) used a fixed-free Stokoe-type resonant column apparatus with suction control using the hanging column technique (McCartney et al. 2008) to measure the value of \( G_{\text{max}} \) for the same sand used in this study, albeit at a relative density of 50%. Although the strain level in the resonant column test was not measured directly, a parametric evaluation indicated that the value of \( G_{\text{max}} \) was constant for the lowest strain values. The strain levels in resonant column test are likely larger than those in the bender element test, but both are within the small strain region (less than \( 10^{-5} \)). A comparison between the measured \( G_{\text{max}} \) values from the bender element (BE) and resonant column (RC) for tests for different degrees of saturation is shown in Fig. 9(a), while the same \( G_{\text{max}} \) data is plotted as a function of the matric suction (inferred from the SWRC of the sand) in Fig. 9(b). Because of the different densities in the BE and RC tests, the \( G_{\text{max}} \) values were normalized using the void ratio function defined by Hardin (1978) \[ F(e) = 1/(0.3 + 0.7e^2). \] Although there is a slight difference in the trends between the two sets of data, the magnitudes are consistent for the measurements from the upper bender element pair and those from the resonant column tests at higher total stress values. The difference in trends between \( G_{\text{max}} \) and degree of saturation obtained from the two tests can be attributed to the slight differences in relative density and net normal stresses, as well as experimental errors in the outflow measurement system for the hanging column test incorporated into the resonant column test used by Khosravi et al. (2010). Despite these differences, this comparison indicates that steady-state infiltration leads to similar stress state conditions as those induced in partially saturated sands in element scale tests such as the resonant column.

**Relationships between \( G_{\text{max}} \) and Mean Effective Stress**

The mean effective stress with depth in the sand layer was calculated by multiplying the vertical effective stress by \( (1 + 2K_0)/3, \)
where $K_0$ is the coefficient of earth pressure at rest. The value of $K_0$ was estimated using a friction angle $\phi$ of 35° for the sand corresponding to a relative density of 45%. The variation in the estimated values of $G_{\text{max}}$ for dry sand with effective stress measured using the bender elements is shown in Fig. 10(a). Results are shown for bender element tests performed during centrifugation at 40 g as well as under 1 g. To check the consistency in the trend of the data for dry sands with mean effective stress, the data was compared with the predicted $G_{\text{max}}$ values from the empirical equations of Hardin and Richart (1963), Seed and Idriss (1970), and Hardin and Drnevich (1972). The values of $A$ and $n$ for these sands were taken directly from their papers, so the predictions were not expected to provide an exact fit. Nonetheless, the values of $G_{\text{max}}$ for dry sand measured by the bender elements are consistent with the trends expected from these relationships.

The results for dry, saturated and partially saturated Ottawa sand are plotted together as a function mean effective stress in Fig. 10(b). This figure contains tests performed at 1 (saturated and dry tests), 40, and 50 g. The mean effective stress for the partially saturated sands was calculated in a similar manner as above, but using Eq. (3) to define the vertical effective stress. Although there is some scatter in the trend of $G_{\text{max}}$ with effective stress for the partially saturated sands, the data tends to follow a unique relationship for dry, saturated, and partially saturated sands. Specifically, a single power function may be used to predict the value of $G_{\text{max}}$ with mean effective stress for Ottawa sand. The data points fall within a tolerance of ±10% around the power function. Because the $G_{\text{max}}$ values for partially saturated sands follow the same relationship with mean effective stress as dry and saturated sands, this observation confirms that steady-state infiltration is an appropriate technique to control the stress state in partially saturated sand layers.

**Conclusions**

This paper describes the details behind a testing program involving the use of bender elements to measure changes in small strain shear modulus, $G_{\text{max}}$, of sand layers during variations in degree of saturation induced by steady-state infiltration in a geotechnical centrifuge. Consistent with results from resonant column tests performed on the same sand with suction control using the hanging column approach, $G_{\text{max}}$ varied nonlinearly with degree of saturation and showed a peak value at a degree of saturation between 0.3 and 0.4. The measured values of $G_{\text{max}}$ of partially saturated sand layers follow the same trend with mean effective stress when the vertical effective stress is defined from the suction and degree of saturation profiles during steady-state infiltration. Accordingly, these observations indicate that steady-state infiltration is an effective tool to control the effective stress in partially saturated sands in centrifuge physical modeling.
Acknowledgments

The writers would like to thank Min Jae Jung and Dr. Kenneth H. Stokoe, II for their assistance in manufacturing the bender elements used in this study, and Kent Polkinghorne for his assistance with the bender element data acquisition system.

References


