1. Find the limits of integration for the integral on the right below so that the order of integration can be reversed without affecting the equality.

\[
\int_2^3 \int_1^4 (1+y) f(x, y) dx dy = \int \int f(x, y) dy dx
\]

2. What is the angle (\(\theta\)) between the contour \(y = 10 - x + 2x^2\) and the force \(F = 2x\hat{a}_x + 3x\hat{a}_y\) at the point (1,1)?)

\(\theta = \ldots \) °

3. If \(A(t) = 10\cos(300\pi t + \pi/4)\) and \(B(t) = 3\cos(300\pi t + \pi/8)\), what is the average value of the product of \(A\) and \(B\)?

\(<A(t)B(t)> = \ldots\)

4. A signal is connected to a three-way splitter as shown. Each of the signals from the splitter is given a different phase shift before being summed. For the phase shifts shown in the figure, determine the magnitude of the output in decibels when compared to the maximum possible magnitude of the output, which would occur when all of the phase shifts are equal.

Output (with phase shifter) = \ldots dB with respect to output with all phase shifts equal.

5. If \(\phi(x,y,z)\) represents temperature(°F), and \(x, y,\) and \(z\) are in feet, how quickly is the temperature at (1,2,1) changing in the direction of \(\vec{V} = \hat{a}_x + \hat{a}_y - 2\hat{a}_z\), given that \(\phi(x,y,z) = (67 + 0.2x - 0.3y - z^2)°F.\)
Answer _______________ °F/foot

6. Calculate the work required to move completely around the contour formed by the intersection of the sphere \( x^2 + y^2 + (z - 3)^2 = 25 \) and the plane \( y = 4 \) if force is given by: \( \vec{F} = 2yz\hat{a}_x - 2x^2yz\hat{a}_y + 4x\hat{a}_z \) Newtons.

Work = _______________ Joules

7. Find the rate at which fluid flows out of a sphere of radius 6, centered on the origin, if fluid velocity is given by:

\[
\vec{V} = 12 \left( \frac{1}{x}\hat{a}_x + \frac{1}{y}\hat{a}_y + \frac{1}{z}\hat{a}_z \right) \text{ m/s}
\]

Flow rate = _______________ m³/sec

8. Sixteen Coulombs per second are leaving a spherical region with a radius of two meters. Assuming that the current density is uniform over the surface, what is the value of the normal component of that current density? Be sure to give units.

\( J_n = \) _______________ (_______)

9. A straight conductor with a circular cross-section of radius \( b \), centered on the \( z \)-axis, carries a current distribution \( \vec{J} \) as defined below. Find the total current flowing on that wire.

\[
\vec{J} = J_0 \left[ 1 - \left( \frac{r}{b} \right)^2 \right] \hat{a}_z \text{ Amps/m}^2
\]

where \( r \) is the radial distance from the center of the wire.

\( I = \) _______ Amps
10. a) Circle the following vector expressions that do not violate the rules of vector algebra:

\[
\begin{align*}
\Phi \nabla \cdot \vec{A} + \nabla \Phi \\
\nabla \Phi \cdot \vec{V} - \Phi \nabla \Phi \\
\nabla (\vec{A} \times \vec{B} \cdot \vec{C}) - \nabla \Phi \\
(\vec{A} \cdot \nabla \Phi) \times \vec{V} \\
\nabla^2 (\vec{A} \times \vec{B} \cdot \vec{C})
\end{align*}
\]

b) Write the equation for a unit vector in the x-z plane that rotates clockwise when viewed from the negative y-axis. That unit vector should rotate with an angular velocity of 60 revolutions per second, and point in the –x direction at \( t = 0 \).

\[\vec{V} = \text{______________________________} \]

c) \( \vec{C} \times \vec{B} \cdot \vec{A} \) is equal to (circle the correct one(s)):

\[
\begin{align*}
-\vec{B} \times \vec{A} \cdot \vec{C} \\
\vec{C} \times \vec{A} \cdot \vec{B} \\
\vec{B} \times \vec{A} \cdot \vec{C} \\
\vec{B} \times \vec{C} \cdot \vec{A} \\
-\vec{C} \times \vec{A} \cdot \vec{B}
\end{align*}
\]

d) Label the coordinate systems on the right as being left handed (L), right handed (R), or undefined (U).

e) Circle the point(s) that lie on the surface defined by \( x^2 + y^2 - z = 8 \)

\[
\begin{align*}
(1,1,-1) \\
(2,2,0) \\
(3,2,5) \\
(0,0,0) \\
(-2,-3,5)
\end{align*}
\]