Capacitor: An Energy Storage Device

When the switch is closed, charges will flow onto the plates of the capacitor. Because the capacitor plates are not touching electrically, current will flow in this circuit only by the flow of charges onto the plate. That will happen only with a changing voltage.

\[
i = C \frac{\text{Change in Voltage}}{\text{Change in Time}} = C \frac{dV}{dt}
\]

where \( C \) is capacitance.

The more rapidly voltage across a capacitor changes, the more current will flow (the higher the frequency, the greater the current).

Units of \( C \left( \frac{\text{Coulombs (stored on plates)}}{\text{Voltage (across plates)}} \right) \) or Farads
One-Conductor Capacitor: because sand particles “scrape” off electrons, the helicopter gets charged to a high voltage
Capacitor: Applications

Capacitors can be used to store energy or to filter signals.

Example: Starting Capacitor—used to provide the extra current needed to start an electric motor.

Example: Filtering out AC—used to get rid of “ripple” from DC power supply.

Example: Filtering out DC—only time-varying signal will pass through capacitor. The circuit on either side of the capacitor can be at different DC voltages.
Capacitor: Applications

Example: **Low-Pass Filter**—as frequency increases, more current will pass through the capacitor and more voltage will drop across the resistor. This will cause higher frequencies to be filtered out.

Example: **High-Pass Filter**
Signal Distortion (also called Frequency Dispersion) Caused by Capacitance

Capacitance and inductance in a circuit allow different frequencies to travel with differing degrees of attenuation and phase shift. This causes the received signal to be different from the signal sent.

In a purely resistive circuit, the signal received will be a scaled version of the signal sent => no distortion.
Sources of Capacitance in Real Circuits

The connection pads on this low-frequency printed circuit board (PCB) create enough capacitance so as to make it unusable at high frequencies.

*Note:* the back side of the PCB is often solid conductor to provide shielding.
Calculation of Connection Pad Capacitance

To determine the frequency limitation imposed by the capacitance of the connection pads shown on the previous slide, we need to estimate its capacitance.

Given that the board is 4 mm (0.004 m) thick, and that its back side is a solid conductor, calculate its capacitance.

Approximate surface area of pad is 35 mm\(^2\) or 35 \(\times\) 10\(^{-6}\) m\(^2\).

The side view looks identical to a parallel-plate capacitor. We will find the capacitance of a parallel-plate capacitor having the plate area and separation as shown above.
Finding the Capacitance of a Parallel-Plate Capacitor

**Assumption**: there is no fringing => the electric field travels in a straight line from one plate to another

Procedure for finding capacitance between any two conductors (remember, we are trying to find the charge stored, $Q$, for an applied voltage, $V$)

1. Assume a charge $Q$ on one conductor, and a charge $-Q$ on the other. Assume that the charge is distributed uniformly on the conductors.
2. Knowing the uniform charge distribution, calculate the electric field between the two conductors.
3. From the electric field, calculate the voltage between the plates. You now know $Q$ and $V$, so you can calculate the capacitance.
Finding the Capacitance of a Parallel-Plate Capacitor (2)

1. If $\pm Q$ is uniformly distributed on the plates, then the charge distribution, $\rho_s$, will equal $Q/A$ Coul/m$^2$

2. The electric field above a large planar conductor with a surface charge distribution is equal to (note that $E$ is non-zero only between plates for this configuration):

$$E = \frac{\rho_s \hat{a}_n}{\varepsilon}$$

Where $\varepsilon$ is the permittivity of the material between the plates = $\varepsilon_r \varepsilon_0$

So $E = \frac{\rho_s \hat{a}_n}{\varepsilon} = \frac{Q \hat{a}_n}{A \varepsilon_r \varepsilon_0}$

The voltage between the plates is given by

$$V = \int_0^d \bar{E} \cdot d\bar{l} = E_n d$$

$$= \frac{Q d}{A \varepsilon_r \varepsilon_0}$$

Thus capacitance $C = \frac{Q}{V} = \frac{Q}{Qd} = \frac{A \varepsilon_r \varepsilon_0}{d}$
Quantitative Results

The capacitance of our connection pad is (letting $\varepsilon_r = 4$) 0.3 pF. To show how this will affect a signal consider putting this capacitance in our idealized circuit.

Ignoring other factors, such as wire inductance and other pad capacitances, and using reasonable values for the components in the above circuit, we can estimate the effect of pad capacitance.
Quantitative Results (2)

For practical values of circuit components \((R_{\text{gate}} = 2 \, \text{K}\Omega, R_{\text{wire}} = 0 \, \Omega, R_{\text{input}} = 2 \, \text{M}\Omega)\), our circuit can reasonably be approximated by:

\[
V_{\text{received}} = \frac{V_{\text{sent}} \left( \frac{1}{j\omega C} \right)}{R_{\text{gate}} + \frac{1}{j\omega C}} = \frac{V_{\text{sent}}}{1 + jR_{\text{gate}} \omega C} \Rightarrow \frac{V_{\text{received}}}{V_{\text{sent}}} = \frac{1}{1 + jR_{\text{gate}} \omega C}
\]

\[
= \frac{1}{1 + j \left( 3.8 \times 10^{-9} \right) f}
\]

This is the transfer function for a low pass filter with a 3 dB cutoff frequency of 250 MHz.
Effect of RC On Signal Propagation

- Idealized Input Signal
- Output With Small RC
- Increased RC
- Greater RC
Surface Mount Components (2)
Copper Interconnects (on integrated circuits)

Copper interconnect wires with dielectric removed:
Just How Small Are These Circuits?

It’s holding a micro-chip
IC speed dominated by RC signal delay in interconnects as IC feature dimensions scale down.