Impact of Effective Stress on the Dynamic Shear Modulus of Unsaturated Sand

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Abstract

The dynamic shear modulus of soils is needed to predict soil behavior in response to cyclic loading. Even though the effective stress has been shown to have a significant impact on the dynamic modulus of water-saturated and dry soils, its effect on the dynamic shear modulus of unsaturated soils has not been evaluated. Specifically, studies on the dynamic response of unsaturated soils have characterized variations in small-strain shear modulus ($G_{\text{max}}$) as a function of the degree of saturation or matric suction alone. In contrast, this study evaluates the use of the suction stress characteristic curve to characterize the impact of mean effective stress ($\sigma'_m$) on the dynamic shear modulus of unsaturated sand. A fixed-free resonant column test device was adapted with a hanging column setup so that the small-strain dynamic shear modulus could be measured for sand specimens under different confining pressures and matric suction values. Trends between the small strain shear modulus and effective stress for unsaturated sand were found to be different from those reported in the literature, where $G_{\text{max}}$ varied linearly with the square root of $\sigma'_m$.

Keywords: Unsaturated, Dynamic shear modulus, Effective stress, Suction stress

Introduction

The dynamic shear modulus is a property of soils used widely to evaluate wave propagation in soils to assess the dynamic response of foundations, pavements, and embankments. Shallow foundations and road subgrades are routinely loaded cyclically by vehicles and machines. In these structures, the soil is rarely water-saturated, but experiences seasonal wetting and drying under unsaturated conditions. A methodology for considering the effective stress in unsaturated soils has precluded use of traditional soil dynamics analysis, such as those for earthquake loading, to
predict strains in unsaturated soils during cyclic loading. Dynamic properties have been evaluated extensively for water-saturated and dry soils (Hardin and Black 1968; 1969; Hardin 1978; Stokoe et al. 2004). Hardin and Black (1969) showed that dynamic properties of soils are dependent on the mean effective stress $\sigma'_m$, void ratio $e$, degree of saturation $S_r$, maximum principal stress difference $(\sigma_1 - \sigma_3)$, soil grain characteristics (shape, size, mineralogy), and gradation. However, for shear strain amplitudes less than $10^{-4}$, dynamic properties are nearly independent of these variables except for $\sigma'_m$ and $e$. The dynamic shear modulus of soils for this range of strain amplitude is referred to as the maximum or small-strain dynamic shear modulus ($G_{\text{max}}$). Hardin and Black (1968) proposed a functional relationship between $G_{\text{max}}$ and mean effective stress $\sigma'_m$, as follows:

$$G_{\text{max}} = 625(\text{OCR})^kF(e)\sqrt{P_a\sigma'_m}$$

(1)

where OCR is the over-consolidation ratio, $k$ is a fitting parameter related to the plasticity of the soil, and $P_a$ is the atmospheric pressure, having the same units as $\sigma'_m$ and $G_{\text{max}}$. $F(e)$ is a function of the void ratio $e$, defined as (Hardin and Black 1969):

$$F(e) = \frac{1}{0.3 + 0.7e^2}$$

Qian et al. (1991) performed one of the first studies to determine the difference in dynamic properties between unsaturated and saturated sands. In their study, the effects of the degree of saturation, confining pressure, void ratio, and grain size distribution of the soil particles on the dynamic shear modulus of an unsaturated sand were measured using resonant column tests. Unsaturated sand specimens were prepared by tamping the sand to reach the same void ratio but with different initial water contents. Their results indicate that the degree of saturation has an important effect on the small-strain shear modulus, especially at low confining pressures $\sigma_c$ [Fig. 1(a)]. It was observed that $G_{\text{max}}$ tended to increase to a peak value at a degree of saturation of 10 to 15%, after which it decreased as the soil became wetter. Specimens that were initially looser (higher initial void ratio) showed a greater impact of the degree of saturation [Fig. 1(b)].

Figure 1: Trends in $G_{\text{max}}/G_{\text{dry}}$ for A4 sand measured by Qian et al. (1991) with confining pressure and: a) Degree of saturation b) Void ratio
Although $G_{\text{max}}$ was observed to follow a clear trend with degree of saturation, a unique relationship was not established. Results indicated that there are parameters other than the degree of saturation affecting the small-strain shear modulus of unsaturated sand. Qian et al. (1991) observed that increasing confining pressures led to a decrease in the void ratio, which led to a decrease in the $G_{\text{max}}/G_{\text{dry}}$ ratio [Fig. 1(b)]. The size of soil particles also decreased the effect of unsaturated component of effective stress on the small-strain shear modulus. Accordingly, the results presented in Figure 1 cannot be re-interpreted to determine the impact of effective stress on the dynamic shear modulus.

As an improvement upon the study of Qian et al. (1991), one of the goals of this study was to evaluate the impact of effective stress state on $G_{\text{max}}$. To interpret the impact of stress state, the concept of suction stress developed by Lu and Likos (2006) can be used to define the effective stress for unsaturated soils, as follows:

$$\sigma' = (\sigma_c - u_a) + \sigma_s$$  \hspace{1cm} (3)

where $\sigma_c$ is the total confining stress applied to the system, $u_a$ is air pressure and $\sigma_s$ is the suction stress of the specimen. The suction stress $\sigma_s$ (or tensile stress) is a function of the matric suction (or volumetric water content), and represents the inter-particle stresses arising from a different phenomena present in unsaturated sands (capillarity, cementation, etc.). The relationship between suction stress and volumetric water content (or matric suction or degree of saturation) is referred to as the Suction Stress Characteristic Curve (SSCC). The SSCC permits use of Eq. 3 to estimate a soil-specific, single-value of effective stress so that engineering analyses developed for saturated soils can be extended to unsaturated soils.

In this paper, unsaturated soil mechanics techniques are integrated with soil dynamics techniques to study the impact of effective stress on the small-strain shear modulus of unsaturated sand. The experiments provide an opportunity to evaluate whether effective stress values defined using failure conditions (such as that shown in Eq. 3) can represent the behavior of soils under small-strain conditions. Specifically, correlation relationships similar to that in Eq. 1 are developed for unsaturated sand.

**Materials**

The soil tested in this study is a silica (quartz) sand with uniform, sub-rounded particles (commonly denoted as F-75 blast furnace sand). The mean particle size is 0.22 mm. The minimum and maximum dry densities are 1469 and 1781 kg/m$^3$, corresponding to maximum and minimum void ratios of 0.8 and 0.49, respectively. The average specific gravity of the sand particles is 2.65. The geotechnical properties of the sand are summarized Table 1. The grain size distribution for F-75 silica sand is shown in Figure 2(a). It can be considered a well graded fine sand. The SWRC for the F-75 silica sand, shown in Figure 2(b), was defined using a hanging column test with controlled outflow, described by McCartney et al. (2008).

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Mineral</td>
<td>Quartz, 99.8% SiO$_2$</td>
</tr>
<tr>
<td>Grain shape</td>
<td>Sub-rounded to rounded</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>2.65</td>
</tr>
</tbody>
</table>
Suction Stress Characteristic Curve

Lu and Likos (2006) defined the SSCC of different soils by extrapolating the failure envelope obtained from triaxial and direct shear tests performed under controlled suction values to the zero shear stress plane. However, Lu et al. (2007) subsequently observed that these tests were not suitable to define the SSCC for sands following this methodology. Specifically, they observed that curvature in the Mohr-Coulomb failure envelopes for sands at small confining pressures caused the tensile strength of the sand to be underestimated. Instead, Lu et al. (2007) used a direct tensile strength apparatus to measure the suction stress of unsaturated sands under different water contents. The tensile strength of unsaturated sand was measured as the force required to split a divided box containing a cubical, unsaturated sand specimen. A direct tensile test was performed by Kim (2001) on F-75 silica sand with the same methodology as Lu et al. (2007). The results from Kim (2001) were re-interpreted using the approach of Lu et al. (2007) to define the SSCC for this sand, as shown in Figure 3. The SWRC for F-75 silica sand is also shown in this figure for reference.

The SSCC shown in Figure 3 indicates that the suction stress is zero for water-saturated sand, consistent with the zero-value of cohesion observed in triaxial testing studies on sands (Bjerrum et al. 1961). The SSCC for sands increases with decreasing volumetric water content (or increasing matric suction) and levels off at approximately 1.2 kPa. The point at which the SSCC levels off corresponds to the air
entry suction. When the soil dries to near residual conditions (*i.e.*, suction values greater than 7 kPa), the suction stress decreases to zero. Although the SSCC is likely path dependent, only the behavior of sands during drying is evaluated in this study.

**Experimental Approach**

A fixed-free Stokoe-type resonant column device with suction-control capabilities was developed at the University of Colorado at Boulder. Figure 5 shows the overall layout of this system. This new apparatus allows control of the total confining pressure and matric suction, two key components of the effective stress of unsaturated soils (Eq. 3). Air pressure is used to impose the cell pressure through a water bath around the specimen while the diffusion of air is limited by placing vacuum grease between a double latex membrane surrounding the soil specimen. The pore air pressure in the soil specimen is vented to atmosphere, so it is considered to be zero. The pore water pressure in the specimen is controlled using the hanging column technique with controlled outflow (McCartney *et al.* 2008). In the hanging column technique, a static column of water is maintained below a water-saturated porous disc (having an air entry suction of approximately 30 kPa). The height of this column of water corresponds to the matric suction at the bottom boundary of the specimen. It is acknowledged that the suction still varies linearly with height within the specimen, but this is currently the best approach to accurately control the matric suction in unsaturated sands due to their low air-entry suction. During testing, increments of matric suction are applied to the soil specimen by lowering the water level in a manometer tube. Outflow with time is measured while maintaining a constant suction value through the use of a Mariotte bottle.

The resonant column test was performed according to ASTM D4015 to determine the small-strain shear modulus G<sub>max</sub>. A swept sine signal with constant amplitude was supplied by a DataPhysics Quattro<sup>®</sup> dynamic signal analyzer to a non-contact electromagnetic drive plate connected to the top of the specimen. The resonant angular frequency of the specimen-drive plate system was measured during application of the swept sine signal using two PCB accelerometers attached to the top platen. The two accelerometers were useful in checking the vertical alignment of the specimen during the test. The average measured resonant frequency was used to calculate the dynamic shear modulus. Because the strain magnitudes imposed in this study were in the linear elastic range (less than 10<sup>-4</sup>%), the same specimen was used to evaluate the impact of different matric suction values on G<sub>max</sub>.

**Specimen Preparation**

Cylindrical specimens were prepared by aerial pluviation (raining) of sand into the membrane expanded over a split mold. Each specimen had a diameter of 35 mm and a height of 70 mm, and a relative density of 50%. The intensity and velocity of raining were controlled by the opening of the funnel from which the sand was poured, and the distance between the funnel and the mold, respectively. This approach was found to provide uniform specimen density. Three specimens were prepared to evaluate the behavior under three values of net normal stress (\(\sigma_c - u_a\)). A total of eighteen testing conditions including six matric suction values under three net normal stresses were considered for the testing program, as summarized in Table 2.
Effect of Matric Suction on $G_{\text{max}}$ of Unsaturated Sand

The SWRCs obtained from the resonant column tests on the three sand specimens evaluated under different net normal stresses are shown in Fig. 5(a). It was found that the SWRCs are relatively the same, with negligible changes in porosity with net stress. A shift in the water retention curve of approximately 1 kPa was noted when the confining pressure was increased from 3.5 kPa to 20 kPa. The small-strain shear modulus values of the sand obtained from the resonant column tests are presented in Figs. 5(b) and 5(c). Similar to Qian et al. (1991), an increase in $G_{\text{max}}$ was noted as the specimen dried from initially saturated conditions. After reaching a matric suction of 4.5 (for tests under $\sigma_c = 3.5$ kPa) or 6 kPa (for tests under $\sigma_c = 12$ kPa and 20 kPa), a decrease in $G_{\text{max}}$ was noted. The degree of saturation for these values of suction corresponds approximately to residual conditions. The effect of matric suction on the small-strain shear modulus of sand can be categorized into three zones (Schubert 1975): a saturation zone ranging from zero suction up to the air entry suction; a transition zone ranging from the air entry suction to the residual suction;
and a residual zone ranging from the residual suction and higher. In the saturation zone, the capillary force is almost negligible and a gradual increase of small-strain shear modulus is observed. In the transition zone the capillary force increases to a maximum value at a low degree of saturation. In this zone, a rapid rate of increase in small-strain shear modulus is observed. In the residual zone the capillary force rapidly decreases toward zero and the small-strain shear modulus decreases with increasing matric suction. This optimum value was found to be close to the water content of the sand at residual conditions.

Figure 5: Results from resonant column tests: a) Points on the SWRC at which G_{max} measurements were made; b) Measured G_{max} values with matric suction and net normal stress; c) Measured G_{max} values with vol. water content

Impact of Effective Stress on Small-Strain Shear Modulus of Unsaturated Sand

The suction stress concept proposed by Lu and Likos (2006) was used to define the effective stress in the unsaturated sand (Eq. 3). Specifically, the suction stress of the soil was obtained using the SSCC fitted to the tensile strength data of F-75 silica sand as shown in Fig. 2(b). A plot small-strain shear modulus data as a function of effective stress computed with Eq. (3) is shown in Fig. 6(a). The magnitudes of small strain shear modulus are consistent with those measured by Pak et al. (2008). Looking at the data as a group, the effective stress is observed to have a pronounced effect on the small-strain shear modulus of unsaturated sand. For each of the confining pressures, the matric suction was varied from 0 kPa to approximately 7.5 kPa. The SSCC shown in Fig. 2(b) indicates that the maximum suction stress observed is 1.1 kPa over this range of matric suction values. When the data from tests under the different values of net normal stress are plotted as shown in this figure, it
appears that the suction stress does not have as significant impact on $G_{\text{max}}$ as the net normal stress. However, for an increase in suction stress of up to 1.1 kPa, an increase of almost 5 MPa was noted in the small-strain shear modulus. Although significant, this is consistent with the observations of Qian et al. (1991).

The lack of a one-to-one relationship between $G_{\text{max}}$ and effective stress defined using the SSCC [Fig. 6(a)] for all of the specimens tested is possibly due to the SSCC assumed for the sand in this study, and may be resolved if the SSCC were redefined using a different approach. Although the tensile test has advantages over other tests, it may not be the most reliable way to define the SSCC for sands because of possible changes in saturation of the specimen after wet-tamping to a certain water content in the split-box setup used by Kim (2001). Another possible source of error is that the hanging column approach does not permit a uniform suction value to be imposed across the entire sand specimen. If anything, the data in Fig. 6 indicates that even small variations in matric suction can lead to a substantial change in shear modulus, so non-uniformity in suction throughout the specimen height may be a major source of error. Although possible, it is not likely that volume change occurred during application of the different matric suction values. Specifically, the shear modulus did not continue to increase, as would have been expected had the soil continued to contract with matric suction. Further, little change in porosity of the specimens was observed under application of different net normal stress values, so it is not likely that matric suction caused densification by a greater amount.

As an alternative interpretation of the effective stress, the same $G_{\text{max}}$ data was plotted against the effective stress calculated by assuming the suction stress was equal to the matric suction [Fig. 6(b)]. This interpretation of the data has some validity because the SSCC shown in Fig. 3 was defined at very large strains. Accordingly, it could have been affected by contraction during shear of the relatively loose sand. This would lead to an effective stress that was not representative of the inter-particle forces as small strains (where $G_{\text{max}}$ was measured). More investigation is needed for higher suction values, as Qian et al. (1991) observed that $G_{\text{max}}$ for dry sand is very close to that of saturated sand. This would then imply that the effective stress defined as in Fig. 6(b) may not lead to as clear of a 1:1 relationship if higher suction values had been imposed.

![Figure 6: Small-strain shear modulus vs. effective stress for F-75 sand: (a) Effective stress from SSCC; (b) Effective stress using $\sigma_s = \psi$](image-url)
The empirical approach to relate the small-strain dynamic shear modulus to the effective stress proposed by Khosravi and McCartney (2009) for compacted soils was extended to the unsaturated F-75 silica sand. The equation was generalized from the equation proposed by Hardin and Black (1969) to include the effective stress defined using the SSCC, as follows:

\[ G_{\text{max}} = A F(e) (\sigma'_m)^n \]  \hspace{1cm} (4)

where \( A \) and \( n \) are fitting parameters defined from the intercept and slope, respectively, of the line that is best fit to a log-log plot of the shear modulus divided by \( F(e) \) versus the effective stress, as shown in Fig. 7. \( F(e) \) is assumed to be the same for both saturated and unsaturated soils according to Eq. 2 (Inci et al. 2003). The same trend is noted in the data whether using two different representations of the mean effective stress shown in Fig. 6.

![Figure 7: Relationship between log\(G_{\text{max}}/F(e)\) and log(\(\sigma'_m\))](image)

The slope and intercept of the linear relationship in Fig. 7 can be used to calculate the values of \( A \) and \( n \), respectively. As the data in Fig. 7 is on a logarithmic scale, the small-strain shear modulus and effective stress must have the same units. A trend between \( G_{\text{max}} \) and \((\sigma'_m)^{0.5}\) was not observed for the unsaturated silica sand. In other words, the effective stress exponents \( n \) are less than 0.5 for silica sand, the value that is typically assumed for saturated or dry sands (Hardin and Black 1968).

**Conclusions**

A resonant column test system was modified to measure the dynamic properties of unsaturated sands. This device has the capability of applying matric suction values ranging from 0 to 10 kPa to the sand specimen using the hanging column technique. The relationship between small-strain shear modulus and effective stress was evaluated for unsaturated sand specimens under different net normal stresses. The effective stress was defined using the concept of the suction stress characteristic curve (SSCC). The small-strain shear modulus was observed to follow a cane-shaped trend with matric suction, and reaches a maximum value at a degree of saturation close to residual conditions (a matric suction approximately 5 kPa). It was observed that the net normal stress had a larger effect on \( G_{\text{max}} \) than the suction stress. A possible reason that a one-to-one relationship between \( G_{\text{max}} \) and effective stress...
was not noted in this study is that the SSCC for the F-75 silica sand was defined using tensile tests. Evaluation of the effective stress by assuming that the suction stress is equal to the matric suction led to a more defined trend between $G_{\text{max}}$ and effective stress. This is possibly due to the fact that the suction stress defined using tensile strength data is representative of large-strain failure conditions, while the matric suction represents the capillarity between particles under negligible or small strain conditions. The overall trends between the drying-path effective stress and $G_{\text{max}}$ for unsaturated F-75 silica sand indicate that established relationships for saturated soils might not be representative of unsaturated sands.

References


