Cyclic Triaxial Test to Measure Strain-Dependent Shear Modulus of Unsaturated Sand

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Abstract: Dynamic shear modulus plays an important role in the seismic assessment of geotechnical systems. Changes in the degree of water saturation influence dynamic soil properties because of the presence of matric suction. This paper describes the modification of a suction-controlled cyclic triaxial apparatus to investigate the strain-dependent shear modulus of unsaturated soils. Several strain- and stress-controlled cyclic triaxial tests were performed on a clean sand with various degrees of saturation. Suction in unsaturated sands increased the shear modulus in comparison with the ones in dry and saturated conditions for different shear strain levels, with a peak modulus in higher suction levels. Also, shear modulus decreased with an increase in the shear strain for specimens with similar matric suction. The normalized shear moduli of the unsaturated sand specimens followed a similar trend to the ones predicted by the available empirical shear modulus reduction functions but showed lower values. The modulus reduction ratios of unsaturated sands shifted up as a result of higher effective stress and suction-induced stiffness. These trends were consistent for both strain- and stress-controlled tests. DOI: 10.1061/(ASCE)GM.1943-5622.0000917. © 2017 American Society of Civil Engineers.

Introduction

Dynamic soil properties are key parameters in the seismic design and performance of geotechnical systems. Accurate estimation of dynamic shear modulus \( G \) is crucial in proper evaluation of soil response to such dynamic loads as earthquakes, blasts, road and rail traffic, wind, and ocean waves. Previous studies indicated that the dynamic shear modulus follows a nonlinear trend where its value decreases as the induced strain level increases (Hardin and Dvhevic 1972).

The small-strain or the maximum shear modulus (i.e., \( G_0 \) or \( G_{\text{max}} \), respectively) occurs at small shear strains \( (\gamma < 10^{-3}\% \) (Kramer 1996). Several empirical equations have been proposed to estimate \( G_0 \) with the use of data from laboratory experiments (e.g., resonant column or bender element tests) or from geophysical methods (e.g., seismic wave methods) (Seed and Idriss 1970; Hardin and Dvhevic 1972). These equations include the effects of void ratio or density, stress history, plasticity index, soil type, and mean effective stress, and they follow a general format presented by Hardin and Black (1969), shown in Eq. 1.

\[
G_0 = A (OCR)^K f(e) P_{at}^a p^n \gamma^n
\]

where \( A \) and \( n \) = fitting parameters that vary for different soils; OCR = overconsolidation ratio; \( p' \) = mean effective stress; \( P_{at} \) = atmospheric pressure; \( K \) = hardening parameter related to the plasticity index of soils \( P_I \); and \( f(e) \) = a function of void ratio.

Recent advancements in unsaturated soil mechanics have revealed the clear influence of the degree of water saturation on the dynamic properties of soils (Khosravi et al. 2010, 2016; Ghayoomi and McCartney 2011). This is a result of the presence of interparticle suction forces that change the effective stresses in soils (Khalili et al. 2004) and, in turn, the mechanical response. Thus, soils with different degrees of saturation differ in stiffness, and this consequently results in different dynamic response, seismic compression, and pore-water pressure generation and dissipation in geotechnical systems (Ghayoomi et al. 2013; Ghayoomi and Mirshekari 2014; Mirshekari and Ghayoomi 2017; Cary and Zapata 2016). Experimental difficulties, such as direct measurement and control of matric suction, have hindered the investigation of the dynamic behavior of unsaturated soils. Although researchers have increasingly explored the effect of degree of saturation on the dynamic soil modulus and damping, they have mostly focused on the small-strain shear modulus for various suction values or degrees of saturation using bender element or resonant column tests (Wu et al. 1984; Qian et al. 1991; Marinho et al. 1995; Cho and Santamarina 2001; Mancuso et al. 2002; Mendoza et al. 2005; Alramahi et al. 2008; Ng et al. 2009; Khosravi et al. 2010;
Ghayoomi and McCartney 2011; Hoyos et al. 2015). In addition, in some recent studies, researchers investigated the effect of suction on the strain-dependent shear modulus, soil volume change, stress path, pore pressure, and drainage conditions in cyclic triaxial systems (Cui et al. 2007; Craciun and Lo 2010; Biglari et al. 2011; Kimoto et al. 2011; Cary and Zapata 2016). However, the effect of the degree of saturation on soil stiffness for medium to large strain levels still requires further examination, especially in relation to dynamic analysis. Recently, Biglari et al. (2011) used cyclic triaxial tests to show the modulus reduction in unsaturated soils. However, the results were not compared with available modulus reduction formulas to check their consistency with the predicted range regardless of the degree of saturation or testing method. In addition, changes in the matric suction during loading cycles and consequent modulus alterations were not reported. The potential application of such a consistent modulus reduction function could be in soil–foundation interaction problems dealing with shallow unsaturated soils, seismic site response analysis with a variable degree of saturation profile, or the deformation of roads and pavement structures resulting from the fluctuation of water content in the soil layers.

This paper explains the modification and implementation of a cyclic triaxial testing system for dynamic loading in a controlled-suction condition. Changes to the system and testing procedures are presented, followed by verification data. The shear modulus reduction data for dry, saturated, and unsaturated soils are presented and compared. Then, the normalized modulus reduction ratios are compared with available shear modulus reduction curves. In addition, the effect of suction (degree of saturation) on measured modulus and the extent of its effect are presented and discussed. Further, the success of drained or constant-suction triaxial tests on unsaturated sand specimens is examined by evaluating the pore pressure and shear modulus throughout the cyclic tests. The objectives of the paper are threefold: (1) show the development process and verification of a suction-controlled triaxial system, (2) demonstrate how suction affects shear modulus, and (3) examine the consistency of the measured data and other available modulus reduction functions using an effective stress-based approach.

Background

**Dynamic Shear Modulus**

The dynamic shear modulus represents soil stiffness in shear and can be calculated from the slope of the shear stress–strain curve obtained from cyclic tests. Parameters such as strain level, loading pattern, intensity, overburden pressure, and water content can affect these properties (Seed and Idriss 1970; Kramer 1996). Using any element-scale dynamic test, a hysteresis loop is formed by plotting the path of stress versus strain during cyclic loading, as shown in Fig. 1(a). This loop is used to estimate the dynamic properties of the geomaterial (i.e., shear modulus and damping).

The backbone curve, shown in Fig. 1(b), which forms a basis for the stress–strain response, is defined by two major values: the steepest slope at small strain \( G_0 \) and the asymptote at large strain (i.e., shear strength, \( \tau_{\max} \)). Secant shear modulus \( G_{\sec} \) (called shear modulus, \( G \), in this paper) at any given strain level is defined by the slope of the line connecting the origin to the point of interest on the backbone curve \( G = G_{\sec} = \tau/\gamma \) and decreases by increasing the shear strain. Although \( G_0 \)-slope is visible in the hysteresis loops obtained from cyclic triaxial tests, it is rarely calculated because of the low accuracy of cyclic triaxial systems in the small-strain ranges. However, a resonant column device or bender element system can be used to determine \( G_0 \). For example, Seed and Idriss (1970) proposed the following empirical equation for the \( G_0 \) of sands:

\[
G_0 = 1,000K_{2,max}(\sigma'_m)^{1/2}
\]  

(3)

where \( K_{2,max} \) is related to the soil relative density or void ratio; and \( \sigma'_m \) = mean effective stress in psf.

The strain-dependent shear modulus declines as the shear strain amplitude rises. In geotechnical engineering practice, this reduction is shown by the normalized modulus reduction curves, that is, the ratio of the strain-dependent shear modulus to the small-strain shear modulus \( G/G_0 \), shown in Fig. 2. Hardin and Drnevich (1972) used a hyperbolic function to present the shear modulus reduction curve, as in Eq. (2). They estimated the reference shear strain, \( \gamma_r \), using the following equation [Fig. 1(b)]:

\[
\gamma_r = \frac{\tau_{\max}}{G_0}
\]  

(4)

where \( \tau_{\max} \) = shear stress at failure, which depends on the initial state of stress in the soil. Hardin and Drnevich (1972) adapted the concept of failure in pure shear and calculated the shear stress at failure using Eq. (5).

\[
\tau_{\max} = \left\{ \frac{1 + K_0}{2} \sigma'_s \sin \varphi' + c' \cos \varphi' \right\}^2 - \left\{ \frac{1 - K_0}{2} \sigma'_s \right\}^2 \right\}^{1/2}
\]  

(5)

where \( K_0 \) = coefficient of lateral earth pressure at rest; \( c' \) and \( \varphi' \) are the static strength parameters (\( c' \approx 0 \) for clean sand); and \( \sigma'_s \) = vertical

![Fig. 1.](image-url)
effective stress. However, one can determine the shear strength (i.e., stress at failure) under the triaxial loading condition as in Eq. (6):

$$\tau_{\text{max}} = \sigma_c \left[ \tan^2 \left( \frac{45 + \phi'}{2} \right) - 1 \right] \cos (\phi')$$

(6)

where $\sigma_c$ = cell confining pressure.

Further, Darendeli (2001) and Menq (2003) modified the basic hyperbolic form by adding a power $a$ to the shear strain ratio, as in Eq. (7). Based on a set of torsional shear and resonant column test data, they proposed the following empirical correlations to estimate $\gamma_r$ and $a$:

$$\frac{G}{G_{\text{max}}} = \frac{1}{1 + \left( \frac{\gamma}{\gamma_r} \right)^a}$$

$$\gamma_r = 0.12 \cdot C_u^{-0.6} \cdot \left( \frac{\sigma_m}{P_a} \right) 0.5 \cdot C_u^{-0.17}$$

$$a = 0.86 + 0.1 \cdot \log \left( \frac{\sigma_m}{P_a} \right)$$

(7)

where $C_u$ = coefficient of uniformity; $P_a$ = atmospheric pressure; and $\sigma_m$ = mean effective stress. Considering the proposed equations for the small-strain shear modulus and shear modulus reduction functions, one can infer that the effective stress could significantly influence the dynamic soil modulus.

**Effective Stress in Unsaturated Soils**

Soils in nature tend to be fully saturated below the water table and become unsaturated above the groundwater table. In unsaturated soils, the air–water interface causes additional interparticle suction that depends on soil type and grain-size distribution (Lu and Likos 2006). The soil–water retention curve (SWRC) represents a constitutive function between matric suction and degree of water saturation.

Negative pore-water pressure in unsaturated soils results in tensile forces that will increase the effective stress. The proposal of an effective stress formula consistent for dry, unsaturated, and saturated soils has been an area of research for several years. It was initiated by Bishop’s effective stress equation (Bishop 1959), which is as follows:

$$\sigma' = (\sigma - u_a) + \chi (u_n - u_a)$$

(8)

where $\sigma'$ = effective stress; $\sigma$ = total stress; $u_a$ = pore-air pressure; $u_n$ = pore-water pressure; $u_a - u_n$ = matric suction; and $\chi = \frac{G/G_s}{G_{\text{max}}}$.
HAEV ceramic disk glued using LORD (Ellsworth Adhesives, Germantown, WI) AP-134 epoxy adhesive. Water flow was supported by two grooves underneath the disk to increase the area of water penetration. Precise control of the flow rate and pore-water pressure was achieved through the DigiFlow pump developed by GEOTAC (Houston, Texas). The design of this equipment is very similar to a syringe that induces various water flows to the specimen. A solid steel reservoir with a capacity of 80 mL was filled with water and connected to a circular piston. By operating the stepper motor, the piston would slowly move with a threaded rod into the reservoir and induce the flow to the specimen. This flow pump is capable of applying flow rates in the range of $3.96 \times 10^{-6}$ to $7.92 \times 10^{-12}$ m$^3$/s. A pressure sensor with a capacity of 690 kPa was attached to the flow pump. The pump can operate in volume- or pressure-control modes and allows selection of the rate, total volume, and direction of flow. Pressure control allows instant application or ramping that is particularly useful for unsaturated soil testing to prevent damage of the HAEV disk. In this set of tests, the valve to the top of the specimen was kept open to the atmosphere to maintain atmospheric pressure, so the matric suction was only controlled by the negative pressure at the bottom of the specimen. For each value of matric suction, the system was left to equilibrate until the volume of withdrawing water was less than 0.002 mL/min, which could take up to 12 h for sand. The volume of withdrawn water was recorded at each step. This procedure was repeated several times with an increment of 1 kPa in matric suction to build a complete SWRC graph.

**Material**

This study used F-75 Ottawa sand, a fine-grained, uniformly distributed silica sand. The grain-size distribution of this sand is shown in Fig. 4, and the physical properties of the material are listed in Table 1. The specimens were prepared at a relative density of 45% representing loosely packed sand. A set of static triaxial tests on dry sand was performed to estimate the soil friction angle and Poisson’s ratio. The Poisson’s ratio was comparable with values obtained from an empirical relation by Seed and Duncan (1986) (Table 1).

The SWRC for unsaturated F-75 Ottawa sand was obtained using the developed system, as described previously and shown in Fig. 5. The SWRC was compared with the curves from previous
Dry density limits, reference to the middle of the specimen, and the cell pressure was set for unsaturated soils, the pore pressure was decreased to zero with saturation until a B-value higher than 0.95 was achieved. For saturated specimens, the pore pressure was kept constant, and the cell pressure was increased to reach the target effective stresses of 50 kPa.

The sand specimens were prepared using dry pluviation (sand raining). Except for the dry tests, all the specimens were saturated by fine de-aired water from the bottom to the top, followed by back-pressure saturation until a B-value higher than 0.95 was achieved. For saturated specimens, the pore pressure was kept constant, and the cell pressure was increased to reach the target effective stresses of 50 kPa. For unsaturated soils, the pore pressure was decreased to zero with reference to the middle of the specimen, and the cell pressure was set to 50 kPa. Then, the target suction value was applied through the pump until the system equilibrated. This approach is similar to a tensiometric suction-control method, although both tensiometric and axis-translation techniques follow the same concept.

Dynamic loads were applied using 10 cycles of sinusoidal deviator strain or stress with a frequency of 1 Hz. Specimens were loaded with an initial seating stress in the stress-controlled tests corresponding to an equivalent initial seating strain in the strain-controlled tests. The purpose of the seating stress or strain was to avoid tension in the specimen during cyclic loading. Single-amplitude deviator axial strains of 0.01, 0.02, 0.05, 0.1, 0.2, and 0.5% and single-amplitude deviator axial stresses of 40, 60, and 80 kPa were applied sequentially in strain- and stress-controlled tests, respectively, as listed in Table 2. Consecutive tests with different deviator strain or stress levels were performed on dry, fully saturated, and unsaturated sands with suction of 2.5, 3, 3.5, 4.5, 6, and 10 kPa. Adequate time gaps between the tests on one specimen were scheduled to re-equilibrate the pore pressure or suction.

The tests were intended to be performed in the fully drained condition (i.e., constant pore pressure or constant suction) by keeping the drainage valve open to the pump with continuous pressure application. However, relatively fast loading prevented a fully drained condition where some excess pore-water pressure was generated. This so-called partially drained condition was very similar to the field conditions during earthquake loads and physical modeling experiments (Dashti et al. 2010). Thus, the change in water pressure was carefully considered in the data-reduction process and shear modulus calculation.

### Data Analysis Methods

After completion of the cyclic tests, hysteresis loops were plotted using the deviator axial stress–strain data. Young’s modulus was determined by calculating the slope of the axial stress–strain hysteresis loop, as in Eq. (10).

\[
E = \frac{\Delta \sigma_d}{\Delta \varepsilon_a}
\]  

where \( \Delta \sigma_d \) = amplitude of deviator stress; and \( \Delta \varepsilon_a \) = amplitude of axial strain. Kokusho (1980) performed a series of cyclic triaxial tests and stated that the shear modulus for a wide strain rate can be calculated from Young’s modulus using the following equation:

\[
G = \frac{E}{2(1+\nu)}
\]  

where \( E = \) Young’s modulus; and \( \nu = \) Poisson’s ratio. An average Poisson’s ratio was used in this equation, simulating a simplified isotropic linear behavior, which was similarly implemented by other researchers (e.g., Georgiannou et al. 1991; El Mohtar et al. 2013). The average Poisson’s ratio measured from the static test and estimated from the empirical relation was used in this study. Given the shear modulus as the ratio of shear stress to strain, \( G = \Delta \tau / \Delta \gamma \), the following relationship was implemented in the analysis to calculate the induced shear strain:

\[
\Delta \gamma = \Delta \varepsilon_a \cdot (1+\nu)
\]  

The secant shear modulus was determined using the slope of the line that connects the two ends of the loop [Fig. 6(a)]. Except for the cases where a significant soil softening or hardening occurred, the average value of all consecutive loops was considered for further analysis. In the large-strain tests, the soil experienced small permanent deformations that caused soil hardening [Fig. 6(b)]. Further, because of the partial drainage phenomenon explained...
previously, excess pore pressure was generated during the large-strain cyclic tests on saturated and unsaturated soils, resulting in soil softening [Fig. 6(c)]. To reduce these adverse effects in such tests, only the first loop with minimal permanent deformation or excess pore pressure was considered in the modulus calculation.

Fig. 7, for example, demonstrates the variations of negative pore-water pressure (i.e., suction) of unsaturated sand with 2.5-kPa suction, measured using the DPT, after cycles of dynamic loading with 0.2 and 0.5% axial strains, respectively. The maximum recorded change of suction after the first cycle of dynamic tests was less than 6%, which led to a minimal change in effective stress. As the cyclic loading generated excess water pressure, simultaneously, the pump tried to equilibrate suction (i.e., reduce the pore pressure). Thus, the pore-pressure variation followed a nonuniform pattern. Although the system could not perfectly represent a drained condition, the partially drained behavior, in nature, is close to what happens in field conditions.

The observed changes in matric suction during cycles of dynamic loading caused variations in the measured shear moduli throughout the cyclic tests. This is illustrated in Fig. 8 for two tests with 0.2 and 0.5% axial strains on specimens with 2.5-kPa suction (the suction variations are shown in Fig. 7). Similarly, the modulus followed a nonlinear pattern because of the simultaneous effects of fast dynamic loads that led to an increase in pore pressure and the continuous suction adjustment by the pump that dissipated the excess pore pressure. However, the change in shear modulus may not be fully correlated with the matric suction fluctuations because dynamically induced permanent deformation could also change the relative density, resulting in material hardening in certain circumstances. Overall, for cases with significant modulus variations, only the shear modulus of the first cycle was considered for the following analysis.

**Experimental Results and Discussion**

**Effect of Degree of Saturation (Suction) on Shear Modulus**

To demonstrate the effect of suction or the degree of saturation on the shear modulus of unsaturated soils, strain-controlled cyclic triaxial tests with various levels of suction (degree of saturation) were performed. Then, the shear modulus and shear strain were

<table>
<thead>
<tr>
<th>Test type</th>
<th>Suction (kPa)</th>
<th>Degree of saturation (%)</th>
<th>Shear strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain control</td>
<td>0</td>
<td>0.069, 0.14, 0.28, 0.69</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.014, 0.028, 0.069, 0.14, 0.28</td>
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<tr>
<td></td>
<td>2</td>
<td>0.069</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>0.014, 0.028, 0.28, 0.69</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.069, 0.14, 0.28, 0.55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>0.069, 0.14, 0.28, 0.69</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.069, 0.14, 0.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>0.028, 0.14, 0.28, 0.69</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.069, 0.14, 0.28, 0.69, 1.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.14, 0.28, 0.69</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.14, 0.28, 1.38</td>
<td></td>
</tr>
<tr>
<td>Stress control</td>
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<td>0.31, 0.33, 0.50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>0.11, 0.15, 0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>0.12, 0.19, 0.26</td>
<td></td>
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<td>0.04, 0.09, 0.13, 0.26</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.19, 0.24, 0.34</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6. Cyclic hysteresis loops for (a) regular cases, (b) hardening in stress-controlled test, and (c) softening cases

Fig. 7. Suction variation during cycles of dynamic triaxial testing

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calculated from the Young’s modulus and axial strain using Eqs. (11) and (12), respectively. The plots in Fig. 9(a) show the changes of shear modulus with the degree of saturation in tests with 0.14, 0.28, and 0.69% shear strain. These shear strain levels correspond to 0.1, 0.2, and 0.5% axial strain values. Higher shear modulus values were measured for unsaturated sands compared with those of dry and saturated sands under the same strain level, as shown in Fig. 9(a). This verifies the stiffer response of unsaturated sand as a result of the presence of interparticle suction stresses. To better visualize this effect, shear modulus variations were plotted against the suction, related to the degree of saturation through SWRC, and are shown in Fig. 9(b). Greater shear moduli were obtained by increasing the suction level. In some cases, the shear modulus dropped in suction ranges higher than the one corresponding to the residual water content.

Overall, this form of modulus variation is mostly in accordance with previously reported trends in small-strain shear modulus of unsaturated sand [e.g., Ghayoomi and McCartney (2011)] and can be attributed to the expected suction stress pattern in unsaturated sand (Lu et al. 2007). Lu et al. (2010) explained that the stiffness and strength properties of soils are much better correlated with suction stress compared with matric suction. They showed that the suction stress reaches a peak by increasing the matric suction in sands as a result of different interaction mechanisms in an air–water–solid interface system. Similarly, this can result in a peak shear modulus (both small-strain and strain-dependent modulus) in unsaturated soils. The noise in the shear modulus data could be associated with experimental data scatter with respect to variation in initial relative density during sample preparation and oscillation in designated pressures that could affect the stiffness and SWRC.

Fig. 8. Shear modulus variation examples during cycles of dynamic triaxial testing

Fig. 9. Effects on dynamic shear modulus: (a) degree of saturation; (b) suction
Suction Dependency of Shear Modulus

The results in Fig. 9 can be explained such that the increase in suction while keeping the net normal stress (cell pressure) constant increased the effective stress in the soil. Thus, the higher effective stress within the soil specimens resulted in a stiffer shear response. To eliminate this effect and explore the shear modulus variations independent of the influence of the effective stress, the measured shear moduli were normalized by the square root of the mean effective stress, as shown in Fig. 10. This normalization method was implemented because of the common correlation between the shear modulus and the effective stress in sands, such as the one in Eq. (3). The trend in the results indicated that suction increased the modulus beyond the expected increase because of the higher effective stress. Given the limited available data, one can propose a suction ($\psi$)-dependent shear modulus relation as follows:

$$G = A(OCR)^{K} f(e) \sigma_0^{0.5} f(\gamma) f(\psi)$$  \hspace{1cm} (13)

Strain-Dependent Shear Modulus

As discussed previously, it is expected that increasing the induced shear strain level decreases the shear modulus. To validate the consistency of this trend for soils with different degrees of saturation, the measured shear modulus values were compared for varying shear strain levels under similar suction conditions. The shear modulus versus shear strain plots for dry, saturated, and unsaturated soils with suctions of 2.5, 3, 4.5, and 6 kPa are presented in Fig. 11. Similar to what has been reported for dry and saturated soils, the shear modulus was found to decrease nonlinearly, sometimes linearly in log-scale plots, for this range of strain levels with an increase in the shear strain level. This trend was similar for all suction levels.

Shear Modulus Reduction Function

To better understand the soil behavior under cyclic loading, the obtained shear moduli were normalized by the small-strain shear moduli, calculated from Eq. (3) proposed by Seed et al. (1970), and are shown in Fig. 12. The effects of seating stress in the axial direction on the mean effective stress and the effect of suction stress in unsaturated sand on the effective stress attained from Eq. (9) were considered in determining the small-strain shear modulus. To compare these data with the available empirical relations for predicting $G/G_0$, shear modulus reduction curves were estimated based on the model by Darendeli (2001) [i.e., Eq. (7)], the model by Hardin and Drenovich (1972) [i.e., Eqs. (2), (4), and (5)], and the basic hyperbolic model [i.e., Eqs. (2), (4), and (6)]; these are shown in Fig. 12. The mean effective stress and vertical effective stress values in these models were consistently used as the chamber cell pressure (i.e., 50 kPa) to avoid demonstrating several curves. The main purpose was to illustrate these reference curves as a way to verify the pattern and the orders of magnitudes.

Regardless of their saturation levels, the measured values fit well in the range of the curves, especially between the proposed equations by Darendeli (2001) and Hardin and Drenovich (1972). One should consider that these empirical curves are based on numerous experimental results with considerable scatter (Oztoprak and Bolton 2013). Thus, a perfect match would not have been anticipated. The $G/G_0$ values for unsaturated soils are different from those for dry or saturated soils. This is because of the nonlinear variation of both $G$ and $G_0$ in dry, unsaturated, and saturated soils. Accordingly, the shear modulus reduction curves for soils shifted up (or, in other words, shifted to the right) with an increase in suction values as a result of their higher shear moduli. The observed modulus variation pattern is consistent with shear modulus curve patterns where higher effective stresses result in a higher modulus reduction range (e.g., Kramer 1996). Further, Dong et al. (2016) reported that the shear modulus not only depends on the applied stresses but also on the degree of saturation through an inversely proportional relation in sands. Thus, the estimated $G_0$ based solely on the stress level may have overpredicted the actual $G_0$ and consequently resulted in lower $G/G_0$ ratios. In addition, $G_0$ values were approximately estimated from an empirical relation that was developed based on the results from dry sand. Thus, the applied relation may not well represent the response of the tested sand, especially in unsaturated conditions. It should be considered that the suction values for this study are relatively small, so future studies on soils that can retain more water in higher suction would be valuable.

Stress-Control versus Strain-Control Tests

Cyclic triaxial tests can be performed in both strain- and stress-controlled conditions. Stress-controlled tests are more popular because of their simplicity in terms of data analysis and better quality of hysteresis loops. Most of the tests in this study were...
performed in the strain-controlled condition because of its capability to accurately control the induced shear strain. Strain-controlled experiments with similar strain levels could be executed on sands with different suction values. However, a set of stress-controlled cyclic triaxial tests was performed on unsaturated sands to compare the results and monitor any inconsistencies that could arise from the testing procedure. After generating the hysteresis stress–strain loops, the induced shear strain and the soil shear modulus were calculated. Then, the normalized shear modulus values ($G/G_0$) were estimated for the stress-controlled tests; they are shown alongside the strain-controlled data and empirical relations in Fig. 13. The results did not designate any meaningful difference between the two testing methods, indicative of consistent modulus measurement in both tests. The measured modulus from the two testing approaches did not align vertically because the shear strain was measured in the stress-controlled tests, whereas it was directly controlled during the strain-controlled tests. Although both the strain- and stress-controlled tests resulted in relatively similar $G/G_0$ ratios, they may not fully represent the soil response in the pure shear condition. One should note that both empirical relations by Darendeli (2001) and Hardin and Drnevich (1972) are based on data from torsional shear/resonant column tests, which directly simulate shear similar to the field conditions. Consequently, careful attention should be given when these curves and triaxial results are used interchangeably.

**Conclusions**

A cyclic triaxial system was modified for suction-controlled testing to investigate the effect of degree of saturation on the strain-dependent shear modulus of unsaturated sand. The study mainly focused on the sand response in medium to large shear strain levels under
strain- and stress-controlled sinusoidal loads. The results indicated that in unsaturated sand, the shear modulus increased with an increase in the suction level, regardless of the induced strain level. However, this increase was more significant in lower strain ranges. Additionally, the modulus variation trends were consistent with reported trends for the small-strain shear modulus where the modulus starts to decrease after a peak value. From an inspection of the variation of the normalized shear modulus with respect to the square root of the effective stress, it is evident that the influence of the effective stress is insufficient to explain the higher shear moduli in unsaturated conditions. Thus, a direct influence of suction could be incorporated in a conceptual suction-dependent shear modulus equation.

With an increase in the shear strain in the specimens with similar suction values, the modulus decreased nonlinearly. The measured shear moduli ($G$) were normalized to $G_0$ values that were predicted from the integration of modulus empirical relations and suction stress-based effective stress formulas. The estimated $G/G_0$ ratios were found to follow a similar hyperbolic trend as predicted by other empirical shear modulus reduction equations. Partially saturated conditions led to a shift in the shear modulus reduction curves as a result of the higher effective stress and suction. However, the measured values were mostly lower than the average $G/G_0$ range estimated from the empirical relations. This could be attributed to the differences in the shear application methods between the empirical formulas and the current study and the inability of current formulations to correctly predict $G_0$ in unsaturated conditions. Further, the results from stress-controlled tests seem to be consistent with the ones from strain-controlled tests, limiting the effect of the triaxial testing approach on the result. Overall, stress state, interparticle forces, shearing technique, density, and soil physical properties are the major factors in determining the shear modulus of soils.

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**References**


